

“The Flow of Water through Pipes.—Experiments on Stream-line Motion and the Measurement of Critical Velocity.” By H. T. BARNES, D.Sc., Assistant Professor of Physics, and E. G. COKER, M.A., D.Sc., Associate Professor of Civil Engineering, McGill University. Communicated by Professor OSBORNE REYNOLDS, F.R.S. Received November 4,—Read November 24, 1904.

In a brief note published in the ‘Physical Review,’* we have described a thermal method of observing the change from stream-line to eddy motion for water flowing through pipes of different diameters. This method we have recently employed in a series of experiments, which we have carried out on a much larger scale than was previously attempted. The method had several points to recommend it for our particular work, the chief one being the simplicity of the experimental arrangements. The object in undertaking this investigation, for which our method was adapted, was in the interest attached to a study of stream-line flow, where every form of initial disturbance was as completely removed from the water as possible.

The impossibility of heating a column of water uniformly throughout while flowing in stream-line motion has been previously observed.† It was shown that, when water is heated electrically while flowing through a tube of two or three millimetres in diameter by a central wire conductor, the heat is carried off by the rapidly moving stream, which forms a cloak of hot water around the wire, and leaves the walls of the tube almost entirely unheated. If the heat be applied to the outside of the tube, the heated water remains in contact with the inner surface of the tube and the water flows through the centre of the tube at a much lower temperature. It is quite different if the flow is increased sufficiently to break up the stream-lines. In this case, eddy or sinuous motion results, and the distribution of heat throughout the water column becomes uniform.

The change from stream-line to eddy motion can be very clearly observed in a tube heated on the outside, since the temperature of the emerging stream immediately increases when the flow rises above the critical point. The point of change is very sharp, and the disappearance of the stream-lines instantaneous.

At the outset of our work we were guided by the two very important papers of Osborne Reynolds,‡ in which the laws governing the flow of water in pipes and channels are worked out. It is clear

* ‘Physical Review,’ vol. 12, p. 372 (1901).

† H. T. Barnes, ‘Phil. Trans.,’ A, vol. 199, p. 234 (1902).

‡ ‘Phil. Trans., 1882 and 1895.

from a study of the work of Reynolds that the change from stream-line to eddy motion may take place within a wide range of velocities. We may observe the critical velocity in two ways: either by observing the velocity at which the stream-lines break up into eddies, or by obtaining the velocity at which the eddies from initially disturbed water do not become smoothed out into stream-lines in a long uniform pipe. The first change may be at any velocity within certain limits depending on the initial steadiness of the inflowing water, while in the second, the change can take place at only one velocity. It therefore depends on whether we start with initially quiet water or disturbed water what value will be attained. Below what we will call the lower limit of critical velocity or stream-line flow, the stream-lines will form the stable flow whatever may be the condition of the water before entering the pipe. If it be in a disturbed state, a short length of pipe is required before the eddies are smoothed out, but stream-lines finally appear further on and subsequently persist as the stable flow. The greater the initial disturbance in the water, the greater will be the length of pipe probably required before the eddies disappear. The flow at which the eddies persist throughout the entire pipe, however long, indicates that the change has taken place from one kind of flow to the other. This point of change is the true critical velocity, and although the production of eddies at the mouth of the pipe may vary, it will be independent of them. If the initial disturbance, however, is not sufficient to prevent the water from starting in stream-line motion, the critical velocity is raised. The change in flow in this case takes place by the birth of eddies in the pipe itself, and the point of change is in some way related to or limited by the degree of steadiness in the water. In regard to this point, Reynolds says, on p. 955 of his original memoir: "The fact that the steady motion breaks down suddenly shows that the fluid is in a state of instability for disturbances of the magnitude which cause it to break down. But the fact that in some conditions it will break down for a large disturbance, while it is stable for a small disturbance, shows that there is a certain residual stability so long as the disturbances do not exceed a given amount.

"The only idea that I had formed before commencing the experiments was that at some critical velocity the motion must become unstable, so that any disturbance from perfectly steady motion would result in eddies.

"I had not been able to form any idea as to any particular form of disturbance being necessary. But experience having shown the impossibility of obtaining absolutely steady motion, I had not doubted but that appearance of eddies would be almost simultaneous with the condition of instability. I had not, therefore, considered the disturbances except to try and diminish them as much as possible. I

had expected to see the eddies make their appearance as the velocity increased, at first in a slow or feeble manner, indicating that the water was but slightly unstable. And it was a matter of surprise to me to see the sudden force with which the eddies sprang into existence, showing a highly unstable condition to have existed at the time the steady motion broke down."

In this connection it is a matter of interest to note that when all forms of initial disturbance, as well as the disturbing influence of the walls of the pipe, are removed, eddy motion is no longer possible. Such a condition of affairs we have in a jet of water issuing from a circular orifice. It is easy to show experimentally that the beautiful rod-like appearance of the water jet depends on the absolute quietness of the water feeding the jet, and that the jet is immediately broken up by producing eddies artificially. Jets flowing in stream-line motion at a speed of from 20—30 feet per second have been obtained by us from a two-inch orifice.

In our present series of experiments we arranged that our pipes should feed from the same tank from which we produced the jets, in order that we might be sure to have the water in a perfectly quiet state. We could, therefore, be fairly safe in assuming that the birth of eddies was the result of the disturbing influence of the walls of the tube only.

Two methods of study were adopted by Reynolds in his experiments. The first was the method of colour bands, which gave the point at which stream-lines break up into eddies, and which we have called the upper limit of critical velocity, and the second was a method of pressures, where the relation between velocity and pressure was obtained above and below the critical velocity. In the colour-band tests a long narrow tank was used, in which the pipe was placed. A flare, carefully smoothed, was fastened to the inflow end, while the outflow end protruded out of the tank and connected with the waste pipe. A small tube containing coloured liquid was directed just in front of the flare and a thin stream of colour was drawn into the tube with the water. This thread of colour remained intact all along the tube for stream-line motion, but disappeared with great suddenness when the critical velocity was reached. The water in the tank was allowed to become as quiet as possible before beginning the experiment. The rate of flow was calculated from the rate of lowering of the water surface, and the dimensions of the pipe. In the pressure experiments the water was allowed to flow directly from the mains into the pipe. After a sufficient length of pipe to allow of the dying out of the eddies, pressure gauges were attached at a fixed distance apart, and readings of velocity and pressure made. The critical velocity was not observed directly by this method, but since the resistance changes with the change in the flow, from the first power of the velocity to about the

1.7th power, the intersection of the logarithmic homologues gave a method of determining it. Reynolds remarks that for a short distance above and below the critical velocity the gauges became unsteady and no readings could be made.

By means of his experiments Reynolds was able to verify his mathematical deductions, and he showed from three different pipes, together with a comparison with the experiments of D'Arcy for large pipes and with Poiseuille's for small tubes, that the critical velocity varied inversely as the diameter of the pipe. He further showed that the critical velocity followed the viscosity temperature law as deduced by Poiseuille, and, therefore, varied as the viscosity over the density. Similar experiments with three different pipes by the method of colour bands showed that the upper limit followed the same laws approximately. The theoretical equations, however, had to do with the lower limit.

In our experiments on the upper limit of critical velocity with absolutely quiet water, we found that stream-lines were preserved in many cases to very much higher velocities than would be expected from the inverse-diameter law, and that the upper limit falls off slightly more rapidly with rise in temperature than the law of Poiseuille. We do not wish to lay stress on these points, or intimate in any way that we think they question the validity of the theoretical laws which have been so completely worked out by Reynolds. We think, however, that they show the instability of the upper limit of stream-line flow, and how much it depends on forms of disturbance. We have, moreover, made independent determinations of the temperature variation of the lower limit, and find by two different methods that over a wide range in temperature the critical velocity follows the viscosity temperature law accurately.*

The slight deviation from the theoretical law for the variation with temperature of the upper limit which we have observed in the case of two pipes of different diameters is, we think, due to the fact that it becomes more and more difficult, as the temperature rises, to maintain stream-line flow in the unstable region. This would result in the upper limit apparently falling off more rapidly than the temperature formula would allow for.

The question has suggested itself to us that some form of initial disturbance in our tank may have come in as the temperature was raised to cause a greater falling-off in the upper limit than the theoretical. Convection currents might have supplied such a form of disturbance, but the size of our tank and the fact that the mouth of our pipes was always placed at or near the middle, together with

* E. G. Coker and S. B. Clement, 'Phil. Trans.,' A, vol. 201, p. 45 (1903); H. T. Barnes, 'B. A. Report,' Belfast (1902).

the length of time required for the tank to cool down, make it appear to us improbable that convection currents played an important part.

Thermal Method of Measuring Critical Velocity.

In some of our first experiments, we observed the change in temperature in the column of water at the critical velocity by noting the increase in resistance of a platinum wire threaded through the centre of the tube, which was heated on the outside, and our preliminary results showed that the presence of a wire of 6 mils. thickness in a tube of $\frac{1}{2}$ inch in diameter was quite sufficient for our purpose. We found that the point of change in flow could be observed more simply by placing the bulb of a mercury thermometer in the stream of water as it flowed out of the tube. A glass prolongation, of slightly greater diameter and connected carefully to the brass pipe by a specially constructed cone or adapter, enabled the readings on the thermometer to be observed. It was a matter of interest to us to see the sudden way in which the reading on the thermometer indicated the point of change by an almost instantaneous change of reading.

We were fortunate in having at our disposal, through the kindness of Dr. H. T. Bovey, F.R.S., the facilities offered by the hydraulic laboratory, where the large experimental tank, 20 feet high and 25 square feet in area, served admirably for a reservoir. The tank stood on the bed-rock, and was therefore free from disturbance in the rest of the building, and, after the eddies occasioned by filling had died out, the water was in a very quiet state. The water used for the experiments was supplied from the Montreal mains and was quite clear; and every precaution was taken, by repeated cleaning, to have the water and tank clean. The action of finely divided matter in suspension in the water is to cause a breaking-up of the stream-lines, so that it was necessary for us to avoid this form of instability.

Our preliminary experiments, made to test the method before applying it on a large scale, were quite satisfactory, and, after profiting by various trials, the final apparatus took the form of that represented in fig. 1.

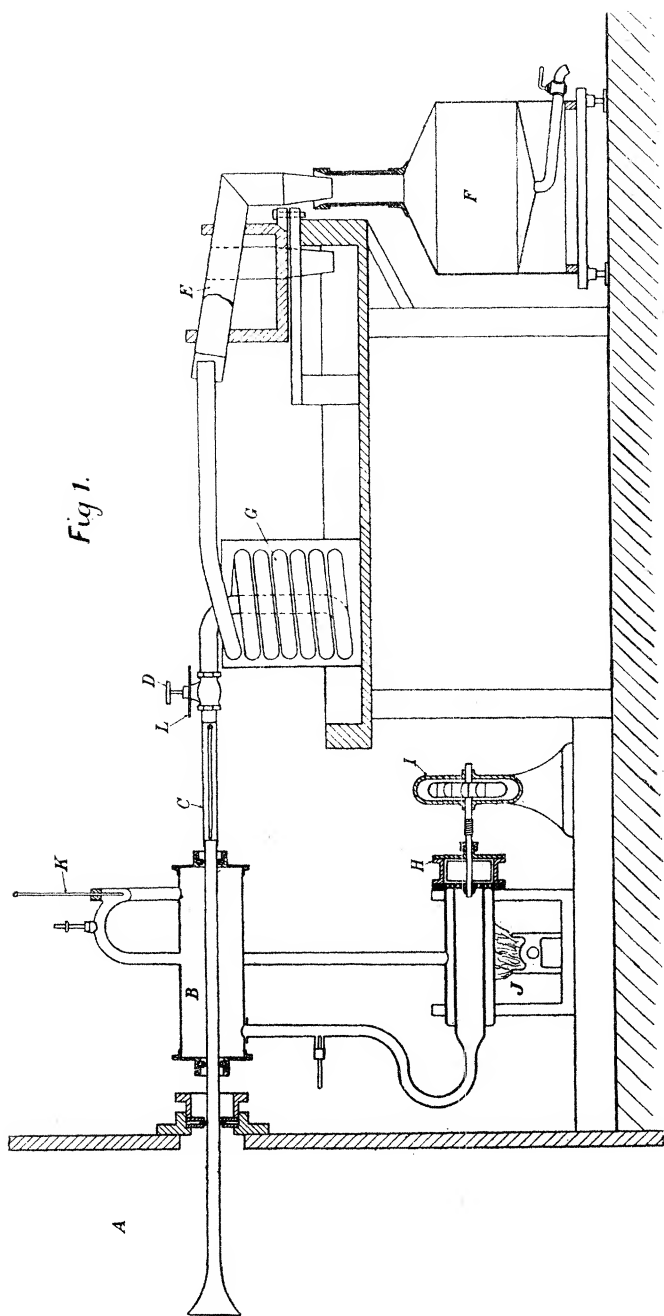
In one side of the large tank A, a hole was cut, in which was fitted a special form of stuffing box to take the tubes of different sizes. Each tube was made of brass and joined to a metal trumpet flare, such that the inside was filed perfectly smooth, and free from sharp points which might disturb the flow. The flare was located at about the centre of the tank, so as to be as free as possible from convection streams along the sides of the tank when working at high temperatures. The end of the pipe protruding from the tank was inserted in a cylindrical water jacket B, with ends closed by stuffing boxes similar

to those on the tank. From the end of the tank a glass prolongation C held the mercury thermometer placed and securely fastened centrally in the tube. The glass tube was cemented into a brass tube, connected with a screw-down valve D, for regulating the flow. From the valve a pipe led to a two-way switch over E, provided with one pipe to waste and one pipe leading to the double-coned copper measure, shown at F. This measure had been very carefully calibrated by Mr. T. P. Strickland some time previously.

For small flows, where the time for filling the tank became inconveniently long, a smaller vessel was used and the amount of discharge obtained by weighing on a large bullion balance. For the high temperature experiments, a worm pipe G was inserted in the outflow so as to cool the water before it entered the measure. Water was circulated continuously through the jacket by means of a centrifugal pump H, operated by a Pelton water motor I. A gas flame J, under the pump, served to regulate the temperature of the jacket. A thermometer K was placed in the water circuit so that the temperature could be regulated at will and maintained a few degrees above or below the temperature of the water flowing out from the tank. A graduated disc L was provided on the valve, by means of which a pointer on the valve gave an indication of the amount opened.

During the greater part of the experiments, the temperature of the tank was taken from a long-stem thermometer placed so that its bulb reached to the centre somewhat below the flare on the pipe. It was found that the temperature thus indicated differed but little from the stream-line reading on the thermometer in the glass prolongation at high temperatures, and was in practical agreement with it at the lower points. For the majority of the experiments, an average head of about 8 feet was maintained, sometimes running down to 4 feet, but seldom getting as low as that. One filling of the copper measure only lowered the head $\frac{3}{4}$ of an inch, so that no constant level device was necessary. The temperature of the tank could be raised to any point up to 90° C. by means of a steam heater. A stirrer was also fitted to the tank, but of course was not kept moving after the temperature of the tank was once brought to a uniform state.

The method of making an experiment was very simple. One observer stood at the valve and regulated the flow until it was seen by the reading that the stream-lines had completely disappeared, and the other observer then switched over the flow into the measure and took periodic readings of the temperatures. The time was taken on a chronograph during the earlier runs, but was changed to a Frodsham and Keen chronometer which had been carefully rated. The water in the tank was always allowed to remain undisturbed for several hours before taking an experiment. For the high temperature experiments the tank was heated to about 90° and then allowed to settle. Over a



week was required for the tank to cool down, during which time successive readings were made. No two determinations were ever made with one setting of the valve, but in every case a re-determination of the critical velocity was made.

It might be questioned whether the presence of the thermometer bulb in the outflowing stream could have produced a disturbing influence, but in no case was the bulb placed inside the tube under test. We think that our very high values obtained in many cases for the critical velocity show conclusively that the bulb had no influence, and especially in the tubes we used, which were all considerably larger than the thermometer bulb.

In order to satisfy ourselves on many points in connection with our measurements, we made some tests to determine the effect of a change of length on the critical velocity. To do this we first took a pipe 3.6 metres long and 1.95 cm. in diameter, and determined the critical velocity by moving the temperature jacket to the end where the glass prolongation began. We obtained the following readings :—

	Temp.	V _c in ft. per sec.	V _c in metres per sec.
	16.0	3.79	1.155
	16.0	3.82	1.163
	16.1	3.66	1.115
	16.1	3.92	1.192
Mean.....	16.05	3.80	1.158

The tube was then removed and cut in the middle, so as to be only 1.8 metres long, and the following readings were obtained :—

	Temp.	V _c in ft. per sec.	V _c in metres per sec.
	15.4	3.66	1.115
	16.7	3.84	1.170
	16.7	3.82	1.163
Mean.....	16.2	3.77	1.148

The agreement is really better than the individual observations. For each of these readings the flow was closed off and then opened carefully until the jump took place on the thermometer. The readings could have been repeated to any desired accuracy of course by leaving the flow unchanged.

The formula for calculating the critical velocity which Reynolds deduced from his experiments for the upper limit reads :—

$$V_c = \frac{1}{43.7} \frac{P}{D},$$

where D is the diameter of the pipe in metres, and P is a function of

the temperature, which was found to be the formula obtained by Poiseuille for the change in flow of water in capillary tubes.

P then equals

$$(1 + 0.03368 T + 0.000221 T^2)^{-1}.$$

Calculating the upper limit for this pipe we find :

$$V_c = \frac{0.0228}{0.0195 \times 1.59} = 0.734.$$

This value comes out 0.734 metre per second instead of 1.15, as we observed it. The higher limit obtained in our measurements is undoubtedly due to the steadiness of the water at the mouth of our pipe.

We convinced ourselves in these experiments that we had not exceeded the upper limit, for the valve controlling the flow was moved very slowly as the point was reached, and stopped instantly the jump took place. It was possible to obtain the stream-lines again by slightly closing the valve. For a velocity of 0.7 metre we had perfectly steady stream-lines. For our smaller pipe, 1.05 cm. in diameter, we found a close agreement with the formula of Reynolds, but that above this, for pipes of larger diameters, the deviations became larger the larger the pipe.

If we tried the experiments too soon after filling the tank, before the water had become perfectly steady, the upper limit came much lower, and, as the water became more settled, a higher limit was indicated by a tendency to form stream-lines, followed by a breaking-up, giving rise to the phenomenon described by Reynolds as "flashing." The flashes became less frequent until, finally, the water being perfectly steady, the higher limit was reached. It was possible to obtain the higher limit as soon as the flashes started, since it was only necessary to increase the flow until the flashing ceased. The flashes showed themselves by an oscillation on the thread of the thermometer.

It was not, therefore, really necessary to have absolutely quiet water to obtain the upper limit, since we were able to satisfy ourselves by experiment that the flashes indicated a disturbance in the tank, which, if removed, would enable us to reach the upper point without their appearance at all. Hence, flashing was taken to indicate a tendency to form stream-lines, and if they appeared, the upper limit was obtained by increasing the flow until the permanent reading on the thermometer indicated that eddy motion was the steady flow.

Stream-Line Flow at High Velocities.

The 1.8 metre length of our brass pipe had remained attached to the tank for several days under an 8-foot head, after completing our experiments, to test the influence of length. On returning to this pipe,

we not only passed our former limit, but were unable to make the stream-lines break up until we produced an artificial disturbance by giving the pipe a sharp rap. The change in flow produced by the jar was seen by the jump on the thermometer reading. It returned to the stream-line reading as soon as the rapping ceased. We, unfortunately, did not measure the flow in this case, but we at least doubled the previous value of 1.148 metres per second. Being somewhat surprised by this result, we made some experiments on this same pipe by colour bands, instead of the thermometer, in order to satisfy ourselves that the high values we obtained were not due to some peculiarity of the thermal method. After some trouble we introduced the colour tube into the tank, and dispensed with the heating jacket. For this we were obliged to let the water out of the tank, and introduce fresh water. Owing to the disturbance thus caused, it was several days before we could proceed with the experiments. It was a simpler matter than we had supposed to observe the colour band, since we had removed the thermometer from the glass prolongation, the thread of colour could be distinctly seen issuing from the brass pipe. We obtained, in these experiments, a critical velocity at the same point as before, but, on opening the valve, the stream-lines re-formed at higher velocities, and persisted to the highest velocity we could produce. This corresponded with our thermal results, except that, in this case, we had an actual re-formation of stream-lines at the higher velocities after eddies had appeared at the usual point.

In order to study the re-formation of the stream-lines more easily, a glass pipe, specially made for critical velocity experiments, with a flare blown on to the tube, was inserted, and a colour tube was put in as before. The tube was 1.2 metres long, and 1.47 cms. in diameter. No thermometer was used, or heating jacket, and the head was about 8 feet, as before. We found, incidentally, that by increasing the quantity of colour it was possible to cause a very large disturbance in the flow.

The following readings were obtained :—

	Temp.	V_c in feet per sec.	V_c in metres per sec.
Little colour	20° C.	3.466	1.056
„ „	20	3.366	1.026
Excess of colour	20	1.252	0.382
Little colour	19	3.786	1.154

These observations showed the importance of using little colour, as a small colour stream produced practically no effect.

Working out the value of V_c as before from the formula, we get at 20° C. the value 0.88 metre per second. This value is, again, much less than the observed value using little colour.

We observed that by opening the valve, and increasing the flow, the stream-lines appeared to re-form. This was shown by a return of the thread of colour. It was not until we reached a velocity of 2.97 metres per second that the thread again disappeared. By altering the flow a little at this point we could make the thread of colour disappear, or obtain it clearly defined. We repeated this several times, and found that the definite nature of this point was remarkable, the thread of colour appearing in almost as sudden a manner as it disappeared.

The diameter of the pipe plays an important part in obtaining the higher stream-line flow, for we found that with a pipe 1.05 cms. in diameter we could not pass the upper limit, nor cause a re-formation of stream-lines. We made some experiments with a brass-pipe, 5.41 cms. in diameter, and 1.5 metres long, by the method of colour bands, and found that we could carry the stream-line flow up to velocities of 1 metre per second, which was the highest flow we could measure. To obtain this flow we were obliged to arrange a much larger measure to handle the water discharged. The upper limit for a pipe of this size, according to the formula, amounts to about 0.24 metre per second at 20° C. We, therefore, exceeded this by four times, as far as we could see, without the formation of eddies. There was a tendency to flash at the highest point, but no definite critical velocity, and the thread of colour could be seen very distinctly.

We cannot enter into a discussion of the influence of the diameter of the pipe on the attainment of the second stream-line flow, but it appears to us obvious that we were able to obtain these higher readings only by paying the strictest attention to the steadiness of the water in the tank. The magnitude of our tank, and the volume of water at our disposal, made this comparatively easy. The inverse diameter law has been shown by Reynolds to be true both for the upper and lower limits, and our experiments show the same where the water has not become perfectly steady; but it is probable that, as the diameter of the pipe becomes larger, the disturbing influence of the walls becomes less effective in causing a breaking up of the stream-lines. In the jet experiments, where there is no directing pipe, stream-line is the stable flow for all velocities, providing the water has become absolutely quiet. For pipes of small diameter, under $\frac{1}{2}$ inch, or 1 cm., the steadiness of the water probably becomes less important, compared with the influence of the walls of the pipe. It is probable that for absolutely quiet water the inverse diameter law holds up to 1 cm., beyond which the critical velocity apparently increases with increasing diameter, until for large pipes we approach the jet. The higher critical velocity may be a second critical velocity, but we have not decided this point.

Temperature Effect on the Upper Limit by the Thermal Method.

In Table I we give the series of experiment obtained for the effect of temperature on the upper limit for our smallest brass pipe, 1.05 cms. in diameter. The results are worked out for each point, correcting the volume of the water in the measure to the volume at the temperature of efflux from the tank. The velocity of efflux was then calculated in the usual manner. In fig. 2 we give a plot of the observations, which extended from 15—86°. Unfortunately, we were unable to extend below 15°, on account of the temperature of the water in the mains not being below that at the outset of the experiments. During the time of the experiments, June and July, after the beginning of the hot weather, the temperature steadily rose in the mains to as high as 20°, where it stayed during the remainder of the work.

Table I.

D = 0.0105 metre.

Temperature.	Total quantity in cubic inches.	Total time in seconds.	V _c in metres per second.
20°.2	3910.7	644.2	1.146
40.1	3898.7	1085.4	0.677
34.1	3891.8	929.9	0.789
29.1	3894.5	811.4	0.905
23.6	3897.9	675.8	1.088
22.8	3895.6	674.3	1.091
56.6	3918.7	1753.0	0.421
49.1	3908.8	1592.7	0.466
84.3	1781.8	245.1	0.137
72.9	1778.1	129.6	0.259
47.3	3893.8	1442.0	0.512
23.3	3899.4	675.5	1.091
20.2	3903.8	643.8	1.149
15.09	3902.9	525.7	1.401

In Table II we give Reynolds' observations for his two smaller pipes, the diameters of which lie just above and below our pipe. These tests were made with colour bands. In the last column we reduce the observations to a pipe equal in area to ours, by the simple ratio of the diameters. In the plot we have included Reynolds' observations, and indicated them by circled crosses. We have, also, included the temperature law of Poiseuille.

Table II.—Reynolds' Observations on Pipes 2 and 3 reduced to a Pipe 1·05 cm. in diameter.

V _c in metres		V _c in metres	
Temperature.	per sec.	Temperature.	per sec.
22°	1·086	6°	1·849
11	1·478	6	1·869
11	1·489	6	1·838
11	1·505	6	1·957
11	1·528	6	1·978
11	1·556	4	1·981
		4	1·891
		4	2·027

In the next table we give a summary of the observations which we obtain for our larger brass pipe at various temperatures.

Table III.

Temperature.	V in feet. per sec.	V in metres per sec.
21°·3	3·458	1·054
34·5	2·059	0·628
40·0	1·791	0·546
51·0	1·362	0·415
55·0	1·115	0·340
70·8	0·631	0·192
73·6	0·777	0·237

These results are plotted in Fig. 2 and represented by circled dots. The general slope of the two curves is the same and, although the agreement between the different determinations is not perfect, it is sufficient to show that the upper limit falls off more rapidly than the theoretical law.

The temperature variation of the upper limit may, from these observations, be represented by the formula

$$P = (1 + 0·0300 T + 0·000704 T^2).$$

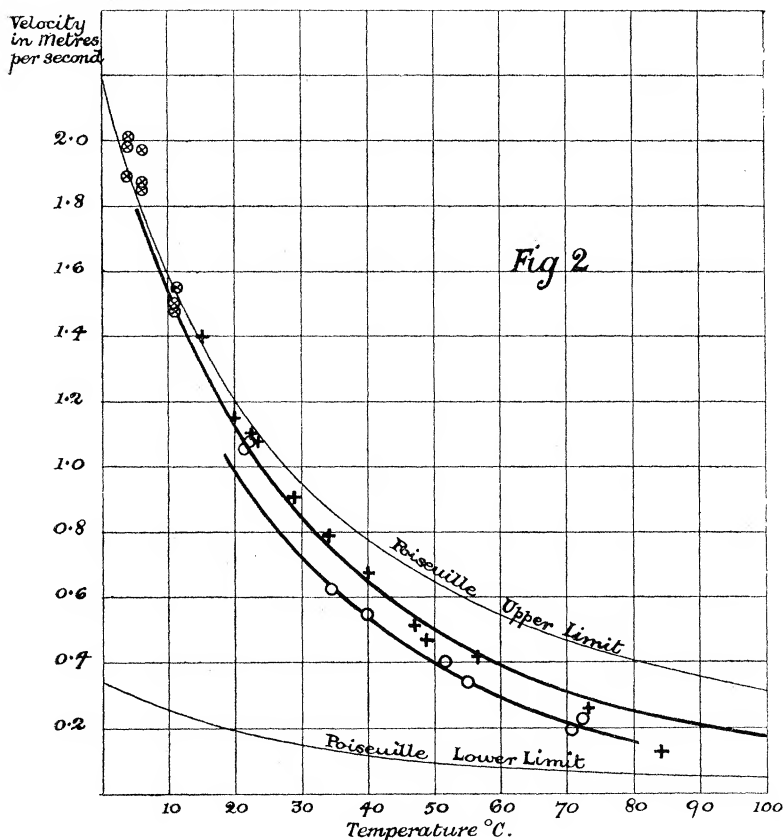
We do not think that the divergence indicates a temperature variation for the critical velocity different to the theoretical, but rather that it shows, as we have pointed out in a previous part of this paper, that the true critical velocity is at the lower limit. The inverse diameter law does not hold for our larger pipe, as will be seen by reference to the plot, and we have shown that it does not hold for the upper limit in the case of our other large pipes.

*Temperature Variation of the Lower Limit.**

In a note by one of the authors (H. T. B.), read before the Belfast meeting of the British Association, it was announced that the thermal

* Compare also E. G. Coker and S. B. Clement, *loc. cit.*

method had been applied to the measurement of the lower limit of stream-line flow, and that, from measurements at different temperatures, it had been found that the variation with temperature followed the theoretical accurately.



The thermal experiments were arranged in the following way:—A large copper tank was fitted with a lead pipe, 8 feet long, coiled up in a smooth spiral. One end of the pipe was connected with the water mains, and the other protruded from the side of the tank and passed through a water-bath, 2 feet long, the temperature of which could be regulated at will and maintained constant by a special form of continuous electrical regulator. The pipe was fitted with a glass prolongation, similar to our previous experiments, and a thermometer was used as before. The tank was fitted with an electrical heater, by means of which the temperature could be changed quickly at will and maintained constant. For the low temperature experiments, an ice-and-water mixture was placed in the tank.

It was found that it was possible to measure three flows: that at which the flow was entirely stream-line with no tendency to form eddies; second, that at which the eddies remained in the flow without the appearance of stream-lines; and third, that at which the eddies and stream-lines followed each other at regular intervals. The change from one flow to the other was observed by the rate of the oscillation on the thread of the thermometer. An inspection of the curves given by Reynolds for the relation between velocity and pressure, above and below the critical velocity, shows that no sharp line of intersection exists between the curve representing stream-line and that representing eddy motion, but that there is a portion over a considerable range of flow where the readings are unsteady. It was in this region that the three flows mentioned above were found. The first was the highest limit of the stable stream-line, the second the lowest limit of the stable eddy flow, and the third the point half-way between. The results are given in the following table for the third point, which is the critical velocity of Reynolds:—

Table IV.
Diameter of pipe, 0·0125 metre.

Temp.	Q. c.c.	Time. secs.	V _c .	P.	UL.
6°	830	30·2	0·225	0·238	0·239
6	866	30·2	0·234	0·238	0·239
17·2	952	45·2	0·171	0·176	0·167
17·2	930	45·0			
18·1	951	45·1			
18·1	926	45·2			
18·1	944	45·1	0·171	0·172	0·163
18·1	960	45·0			
18·1	923	45·2			
40·5	580	45·3			
40·5	568	45·2	0·103	0·104	0·086
40·5	565	45·1			

The first column contains the temperature at which the critical velocity was measured. The second column contains the total quantity of liquid Q which was collected in the measuring glass during the time given in the third column. The times were all taken on a stop-watch. Under V_c are given the values of the critical velocity, calculated in the usual way from the area of the pipe and the flow per second. Under P the values of the critical velocity are given, calculated by Reynolds' lower-limit formula. This formula reads—

$$V_c = \frac{1}{278} \cdot \frac{P}{D}$$

in metres per second, where P and D have the same meaning as before.

In the last column, UL, the values of V_c are given, calculated by the same formula, but using for P the temperature variation deduced from the experiments on the upper limit. The correspondence of columns 4 and 5, and the divergence in column 6, are quite sufficient to show that the temperature variation of the lower limit is in agreement with the law of viscosity rather than with the formula obtained from the upper limit. The experiments are important in showing a close agreement with the theoretical formula of Reynolds, which requires not only the viscosity formula to hold, but the inverse-diameter law as well. The agreement of the individual observations confirms our previous conclusions, in showing the uncertainty in measuring critical velocity, not from the want of accuracy in the measuring appliances, but from the variation in the point itself. It is only by taking a mean of a number of observations that anything like an accurate value can be obtained.

“The *Rôle* of Diffusion during Catalysis by Colloidal Metals and Similar Substances.” By HENRY J. S. SAND, Ph.D., M.Sc., University College, Nottingham. Communicated by Professor J. H. POYNTING, F.R.S. Received November 22,—Read December 8, 1904.

In a paper on reaction-velocities in heterogeneous systems, Nernst* has recently put forward the view that all chemical reactions taking place on the boundary of two phases proceed to equilibrium practically instantaneously, and that the velocities actually observed are simply those with which diffusion and convection renew the reacting material at the boundary. As a special instance of heterogeneous reactions, he mentions catalytic decompositions due to finely divided particles, such as colloidal metals, and he believes it probable that the kinetics of these reactions can be deduced from the assumption that chemical equilibrium remains permanently established on the surface of the particles.

Processes of this kind have recently undergone a considerable amount of experimental investigation, the most important instances being the catalytic decomposition of hydrogen peroxide due to colloidal platinum† and colloidal gold‡ by Bredig and his pupils, and the catalytic decomposition of the same substance due to hæmase by Senter.§ In his last paper Senter discusses his reaction in the light of Nernst's hypothesis, and arrives at the conclusion that the known facts do not contradict it.

* ‘Ztschft. Phys. Chem.,’ vol. 47, p. 52 (1904).

† Bredig and Müller, v. Berneck, ‘Ztschft. Phys. Chem.,’ vol. 31, p. 258 (1899); Bredig and Ikeda, *ibid.*, vol. 37, p. 1 (1901).

‡ Bredig and Reinders, ‘Ztschft. Phys. Chem.,’ vol. 37, p. 323 (1901).

§ *Ibid.*, vol. 44, p. 257 (1903); ‘Roy. Soc. Proc.,’ vol. 74, p. 201 (1904).